# The Liberating Quality of Open, Creative Mathematics 

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In our work as mathematics educators providing tasks and resources on youcubed. org and using the tasks in classes with students, we have found that certain tasks have a liberating quality. When people work on them, they start to see mathematics differently and they start to see themselves differently. This article highlights some of these tasks and our design principles in writing or choosing the tasks. Some of these tasks have brought teachers to tears in our workshopstears of happiness and relief as they realize that they can learn complex mathematics, and see connections they had never before seen. (For more details, see the video, "Believe in Your Maths Potential at tinyurl.com/jotedxoxford.)

The tasks have a similarly liberating quality when used with younger students. In a recent summer camp, we taught middle school girls from under-resourced backgrounds and invited the students to explore visual, creative tasks. We encouraged students to see problems in different ways, to be creative, and to value struggle and mistakes. At first this was difficult for the students - they were so used to engaging narrowly in mathematics. We found that they were only willing to raise their hand if they knew an answer, and they were not willing to share ideas. It was clear that they were not used to teachers asking them to see things differently or to share different ways of thinking and seeing mathematics.

Over the next several lessons, we worked to unlock the students. We shared mindset messages, and importantly, we also continued to invite them to engage differently with mathematics. After many years of working with students of different ages and backgrounds we have come to believe that there are two completely different versions of mathematics. We think of one - the mathematics most people know - as performance mathe-
matics. The other version we describe as mathematical freedom, which is described more fully in Figure 1 below and here (with posters): www.youcubed.org/resource/mathe-matical-freedom/.

## Two Forms of Mathematics

Performance Mathematics
Repeat a method shown
by the teacher
quectice with similar
questions
You will be judged
according to how quickly
and accurately you
reproduce the methods
Mathematical Freedom
$\checkmark$ Consider a problemwhat mathematical idea could you use to solve it?
$\int$ What connections are there between ideas?

Figure 1
As time went by, the students became unlocked, and when they did, their mathematical contributions and thinking were unlike any we had previously experienced. The students asked creative questions, they extended problems into new realms, and they became "limitless" (Boaler 2019). A short film documents the change experienced by the students (www. youcubed.org/resources/math-camp-2019/).

In the remainder of this article, we consider the nature of these important "liberating" tasks.

## Design Feature 1: Can the Mathematics Be Seen and Approached in Different Ways?

There are many rich mathematical tasks that invite students to see and explore mathematics in different ways (see youcubed.org/tasks/ for examples). But, teachers can also make small changes to closed and narrow questions that turn them from closed into open questions. Compare the following examples:

1. Closed: Find the area of a 4 in. by 6 in. rectangle.

Open: How many different rectangles can you draw with an area of 24 square inches?
2. Closed: What is the perimeter of a 5 inches by 6 inches rectangle?

Open: What are the differences and similarities between rectangles that have an odd number perimeter or an even number perimeter?
3. Closed: Number calculations e.g. $12 \times 3$, $148+89$.

Open: Use number talks where students are encouraged to think about number questions mentally, and then share different strategies and visuals. Figure 2 below shows examples for $18 \times 5$.




Figure 2
Encourage the use of visual representations to enhance number talks. We begin this type of number talk by first sharing visuals that go with different methods. Later we ask students to make visual representations for their different methods. When students relate numbers to visuals, it creates important new brain connections (Boaler 2019).
4. Closed: Write the equation of a line that passes through the points $(0,5)$ and $(-3,0)$.

Open: "What functions can you create that pass through the points $(0,5)$ and $(-3,0)$ ?"

## Design Feature 2: Open Tasks So They Are Visual and Allow for Individual Creativity.

One of our favorite ways to open a task in this way is to first ask, "How do you see it?" If students are not familiar with visualizing mathematical ideas, manipulatives can support their learning journey. We love to use dot card number talks. These are an excellent way for students to understand that it is possible to see all of mathematics in many different ways-even seven dots. In one of our teaching experiences with middle school girls we asked them how they saw this pattern (Figure 3):


Figure 3
They amazed us by sharing 24 different ways. See Figure 4 Below:


Figure 4
Dot card number talks support students in understanding that there are many different ways to see patterns. It is important for students to develop their own visuals, and not only to see examples of visuals given by a book or teacher. This is a strategy that we use with all ages of learners. Parents and other adults enjoy the experience, too. For more information on dot card number talks, follow the link for an example: youculbed.org/re-
sources/jo-teaching-visual-dot-card-numbertalk/.

The Border problem (Boaler and Humphreys 2005) is a beautiful task that helps students understand and appreciate the connections between numerical expressions, visual representations, and variable expressions. The task begins by telling students that you are going to show them a $10 \times 10$ square that has a red border. Their goal is to figure out how many squares are in the red border without counting the individual red squares. When we do this task with students, we only show the image with the $10 \times 10$ square with a red border for a few seconds. We do this because we do not want students to count the squares. We then collect ideas from students about how many squares are in the border and how they saw it. Figure 5 is a representation of the different ways our summer camp students saw the border - which, later, developed into number sentences and variables. We share this and other visual algebraic lessons in youcubed's first four weeks of algebra lessons found at youcubed.org/algebra/.


Figure 5
The most important support you can give students is to value all of their creative thinking and give them the space to discuss and share their ideas.

## Design Feature 3: Tasks Encourage Pattern Seeking and Reasoning.

The human mind seeks and finds pleasure in patterns. Young children love rhymes, music, and playful games that are iterative. As mathematical thinkers, we seek to find patterns in order to make sense of a task. One of our favorite pattern tasks is one based upon the Collatz Conjecture (Figure 6). This unsolved mathe-
matics problem has lovely number patterns that excite students. We have shared this task with teachers and students from 3rd grade upwards and have enjoyed the stories of student excitement and perseverance. Teachers have shared with us some beautiful representations their students have created. Keely Hulme shared this one with us (Figure 7).

## The Collatz Conjecture

$\checkmark$ Start with any whole number
-If the number is even, divide it by 2 (or have it)
$\checkmark$ If the number is odd, multiply it by 3 and add 1
Continue generating numbers until your sequence ends
Choose another number and create the sequence
$\mathcal{\sim}$ What patterns do you notice? Can you illustrate your pattern visually?

Figure 6


Figure 7
To download the task go to youcubed.org/ wim/oh-hail-the-elephant-6-8/ It is also available in 3-5 and 9-12 versions.

Pascal's Triangle is another occasion for student wondering and pattern finding. When we shared the magical triangle with our camp students, we asked them fill out the missing numbers in the chart and then to look for patterns by color coding any they found (see Figure 8). We reminded them of the rich cultural history of the triangle and that there may be some patterns no one has yet discovered. Our students enjoyed this pattern-seeking activity and happily shared their conjectures with their peers.


Figure 8
You can find this task at youculbed.org/ wim/pascals-triangle-6-8/ (also available in the grade $3-5$ version). For an extension activity that pushes this task into more creative pattern work see youcubed.org/wim/messing-with-pascal-6-8/ (also available in the grade 3-5 version).

## Conclusion

If we want students to develop growth mindsets, it is essential to open tasks so that they have space inside them for learning. When students see fixed content-short questions with one answer - they do not see how they can grow and learn, and growth mindset messages fall flat. You may be concerned about these recommendations since you have textbooks to work through given to you by others. Our advice-if you are in this constrained sit-
uation-is to work out the big ideas of your year or course and for each big idea choose rich, creative, and deep tasks. We shares our approach to big ideas through a $\mathrm{K}-8$ book series and a free paper called Mindset Mathematics (see youcubed.org/ resource/k-8-curriculum/).

Faced with a page of repetitive textbook questions, our advice is to eliminate most of them and open up the rest so that students can engage collaboratively and deeply.

Youcubed recently launched a new initiative to encourage a data science perspective $\mathrm{K}-12$. There is a free online course on the ideas (youcubed.org/21st-century-teaching-and-learning/) and a range of lovely data visualizations that you can show to students (youcubed.org/resource/data-talks/). Similar to a number talk, a "data talk" invites students to consider:

## What do I see? <br> What is interesting to me? <br> $\int$ What questions do I have?

Our final advice to anyone who wants to take a visual, creative approach is to enjoy the mathematical ideas yourself. Adopt a growth mindset and believe in your own potential. Experience the liberating quality of open mathematics tasks. You may never think about mathematics - or yourself-in the same ways again.

## References

Boaler, J. 2019. Limitless Mind: Learn, Lead, and Live Without Barriers. New York City: HarperCollins.

Boaler, J. and C. Humphreys. 2005. Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning Vol. 1. Portsmouth, NH: Heinemann Educational Books.

## Are You Registered For the Conference?

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